Tutorial 4: Questions

February 7, 2018

Finding a MLE: Poisson

Suppose that the number of Legionella bacteria in a 1 litre sample of water follows a Poisson distribution with unknown parameter λ. Given a random sample $X_1, X_2, \ldots, X_n$

(a) Derive the MLE of λ. Is it biased or unbiased?

(b) Suppose we are given the following observations:

232	225	249	233	242	203	223	229	224	230	235	217	217	192

Calculate a maximum likelihood estimate for λ.

Question 6.2.20, Page 273

A diagnostic test for a certain disease is applied to n individuals known to not have the disease. Let X be the number among the n test results that are positive (indicating presence of the disease, so X is the number of false positives) and p be the probability that a disease-free individual's test result is positive (i.e., p is the true proportion of test results from disease-free individuals that are positive). Assume that only X is available rather than the actual sequence of test results.

(a) Derive the maximum likelihood estimator of p. If $n = 20$ and $x = 3$, what is the estimate?

(b) Is the estimator of part (a) unbiased?

(c) If $n = 20$ and $x = 3$, what is the MLE of the probability $(1-p)^5$ that none of the next five tests done on disease-free individuals are positive?

Question 6.2.28, Page 273

Let $X_1, X_2, \ldots, X_n$ represent a random sample from a Rayleigh distribution with density function:

$$f(x;\theta) = \begin{cases} \dfrac{x}{\theta} e^{-x^2/2\theta} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Derive the maximum likelihood estimator of θ, and then calculate the estimate for the vibratory stress data given below.

16.88	10.23	4.59	6.66	13.68
14.23	19.87	9.40	6.51	10.95

(b) Derive the MLE of the median of the vibratory stress distribution. [Hint: First express the median in terms of θ.]

Question 6.S.32, Page 274

Let $X_1, X_2, \ldots, X_n \stackrel{\text{iid}}{\sim} \text{Unif}(0, \theta)$.

(a) Show that the MLE for θ is $\widehat{\theta} = \max(X_i)$.

(b) Find the CDF and PDF for $\widehat{\theta} = \max(X_i)$. Show that the estimator in (a) is biased.

(c) Find an unbiased estimator for θ.